1.1 Vectors and Vector Operations.

- **vectors** - ordered lists of numbers or variables.
- **dimension** of a vector - number of entries in the vector.
- $v = (2,3) \in \mathbb{R}^2 \Rightarrow$ 2 dimensional vector, is an element of $\mathbb{R}$.
- **head** - front of a vector ; **tail** - opposite end.
- **coordinates** of a vector specify its length and direction but NOT its location in space.
- **standard position** - when tail at origin.
- $v + w = (v_1 + w_1, v_2 + w_n, \ldots, v_n + w_n)$
- $v + w = w + v$
- $(v+w) + x = v + (w + x)$
- $cv = (cv_1, cv_2, \ldots cv_n)$
- $c(v+w) = cv + cw$
- $(c+d)v = cv + dv$.
- $c(dv) = (cd)v$.
- linear combination - $c_1v_1 + c_2v_2 + \ldots + c_kv_k$.
- standard basis vectors - all entries 0 except for a single entry which is 1.
- every vector v can be written as a linear combination of the standard basis vectors,
- dot product → increase when v & w closer, decrease when away,
- $v \cdot w = w \cdot v$
- $v \cdot (w + x) = vw + v \cdot x$.
- $v \cdot (cw) = c(v \cdot w)$.
- vector length, a.ka. euclidean norm aka norm.

- $\|v\| = \sqrt{v \cdot v} = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$
- $\|cv\| = |c| \, \|v\|$
- $\|v\| \geq 0$ equality if and only if $v = 0$.
- unit vectors - vectors with length equal to 1.
- unit vectors arise in situation where the vector's direction is important, but not length.
- normalization - rescaling vector to have length 1.
- $v = \|v\| u$, $u = \frac{v}{\|v\|}$ → unique when $v \neq 0$

  u is not unique.
- Cauchy-Schwartz: $|v \cdot w| \leq \|v\| \|w\|$
- Triangle inequality: $\|v + w\| \leq \|v\| \, \|w\|$